Quark localization and spectral correlations in Lattice QCD

Project Proposal

L. Holicki, L. von Smekal, E.-M. Ilgenfritz

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JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

Joint Institute for Nuclear Research
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Outline

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Metal-Insulator transition in dirty lattices

- P. W. Anderson: Nobel prize 1977
- vanishing $T = 0$ conductivity at high impurities

$$H = \sum_i \epsilon_i |i\rangle\langle i| - \sum_{i,j} t_{i,j} |i\rangle\langle j|$$

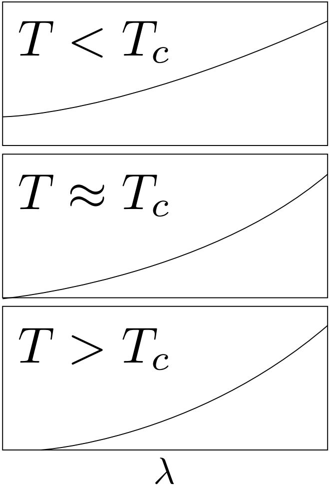
PHYSICAL REVIEW VOLUME 109, NUMBER 5 MARCH 1, 1958

Absence of Diffusion in Certain Random Lattices

P. W. ANDERSON
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received October 10, 1957)

This paper presents a simple model for such processes as spin diffusion or conduction in the "impurity band." These processes involve transport in a lattice which is in some sense random, and in them diffusion is expected to take place via quantum jumps between localized sites. In this simple model the essential randomness is introduced by requiring the energy to vary randomly from site to site. It is shown that at low enough densities no diffusion at all can take place, and the criteria for transport to occur are given.

The Banks-Casher relation



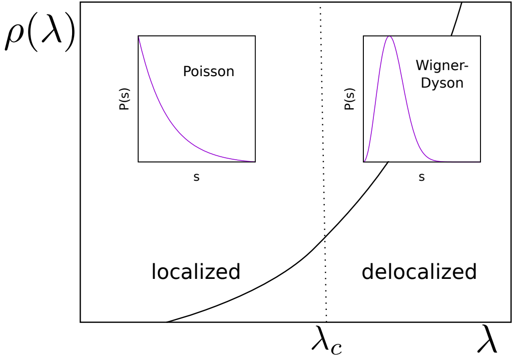
Low modes dominate quark propagator:

$$S(x,y) = \sum_j \frac{\psi_j(x)\psi_j^\dagger(y)}{\lambda_j + m}$$

Spontaneous χSB:

$$-\langle\overline{\psi}\psi\rangle \equiv \Sigma = \pi \lim_{\lambda\to 0} \lim_{m\to 0} \lim_{V\to\infty} \rho(\lambda)$$

Quark localization in QCD



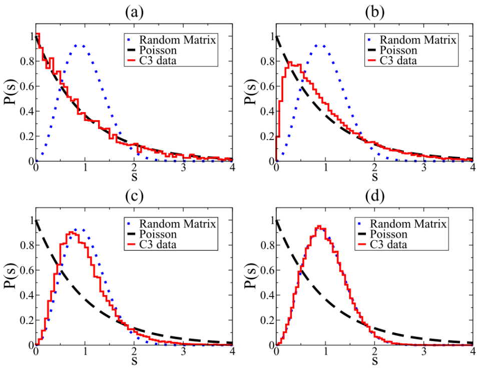
$$P_{\text{Poisson}}(s) \to P_{\text{RMT}}(s)$$

$$s_j = \frac{\lambda_{j+1} - \lambda_j}{\langle \lambda_{j+1} - \lambda_j \rangle}$$

Dyson index	RMT dof.	Ensemble
$\beta = 1$	real	chGOE
$\beta = 4$	quaternion real	chGSE
$\beta = 2$	complex	chGUE

Previous research

- Gavai, Gupta, Lacaze, Phys. Rev. D.77.114506 (2008)
- Kovacs, Phys. Rev. Lett. 104.031601 (2009)
- Kovacs, Pittler, Phys. Rev. Lett. 105.192001 (2010)
- **Kovacs, Pittler, Phys. Rev. D.86.114515 (2012)**
- Bazavov et al. (HotQCD), Phys. Rev. D.86.094503 (2012)
- Cossu et al., Phys. Rev. D.87.114514 (2013)
- Giordano, Kovacs, Pittler, Phys. Rev. Lett. 112.102002 (2014)



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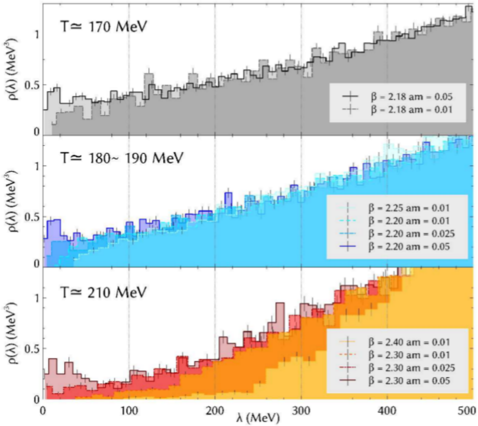
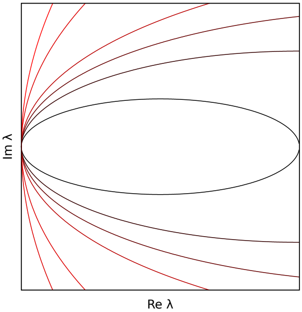


Chiral symmetry on the lattice

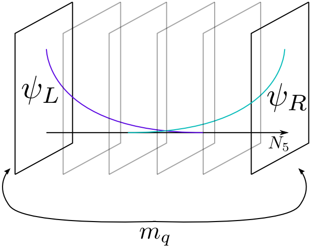
Ginsparg-Wilson eq.:

$$\{D, \gamma_5\} = aD\gamma_5 D$$

$$\Rightarrow \text{Re}\ \lambda = \frac{a}{2}|\lambda|^2$$



Domain-Wall fermions:



$$N_5 \to \infty$$
$$m_5 \to 0$$

Overlap fermions:

$$D^{ov}_{i,j} = \left(\rho - \frac{m_q}{2}\right)\left(1 + \text{sgn} D^W_{i,j}\right) + m_q$$

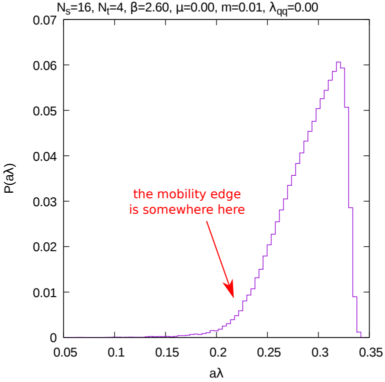
$$\text{sgn}(H) = \frac{H}{\sqrt{H^\dagger H}}$$

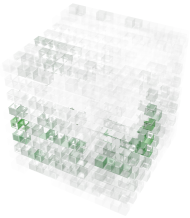
Outline

reproduction of

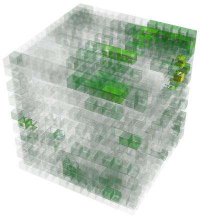
Kovacs, Pittler, Phys. Rev. Lett. 105.192001 (2010)

$V = 16^3 \times 4$, $T \approx 2.6\,T_c$, Quenched, Staggered operator





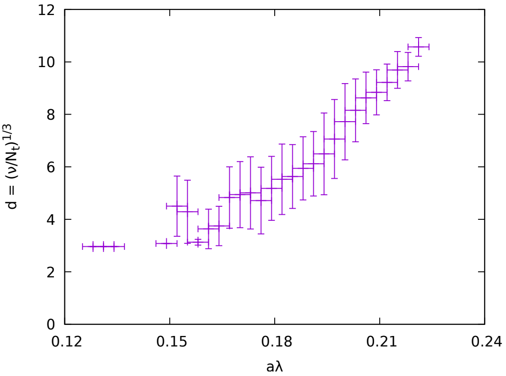
$a\lambda = 0.21415$



$a\lambda = 0.25288$

reproduction of

Kovacs, Pittler, Phys. Rev. Lett. 105.192001 (2010)



Inverse Participation Ratio (IPR)

$$\nu^{-1} = \sum_x (\psi^\dagger(x)\psi(x))^2$$

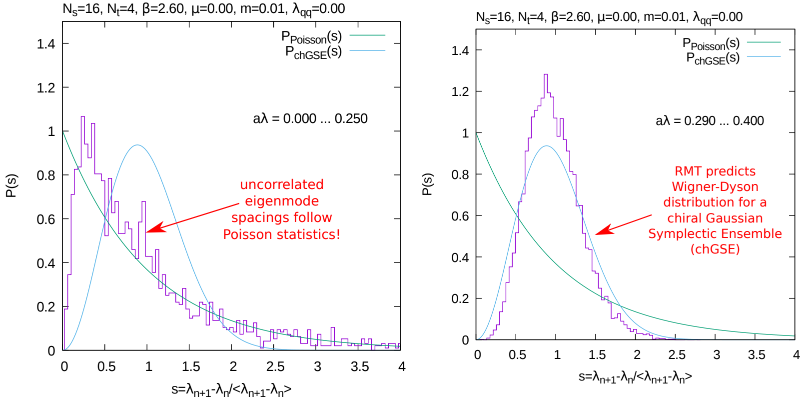
Eigenvector "volume"

$$d = (\nu/N_t)^{1/3}$$

reproduction of

Kovacs, Pittler, Phys. Rev. Lett. 105.192001 (2010)

Unfolded Level Spacing Distribution (ULSD)



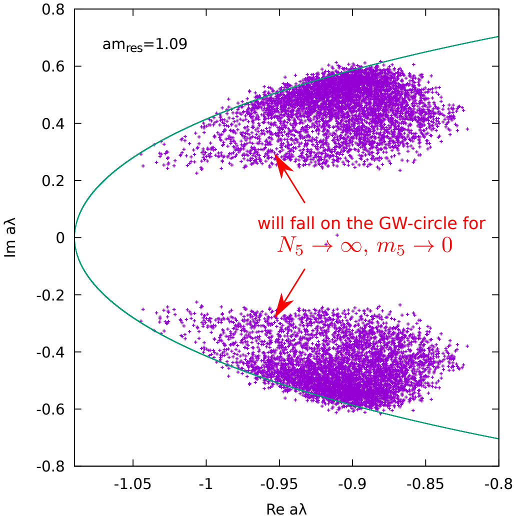
$$s_j = \frac{\lambda_{j+1} - \lambda_j}{\langle \lambda_{j+1} - \lambda_j \rangle}$$

Outline

$V = 10^3 \times 8 \times 16$, $\beta = 1.7$, $am_q = 0.01$, $am_5 = 0.01$

$N_f = 2$ dyn. DW-Quarks, Symanzik imp.

$$\langle P \rangle = -0.233(1)$$
$$\langle z_2 \rangle = 0.393(1)$$
$$\langle l \rangle = 0.001(7)$$



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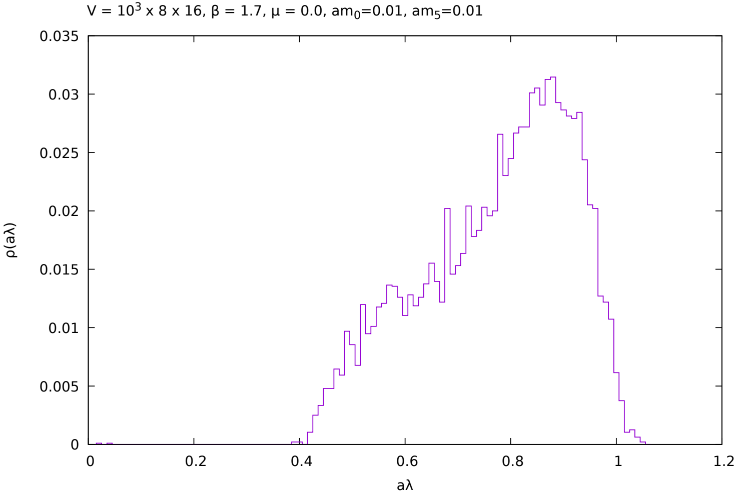

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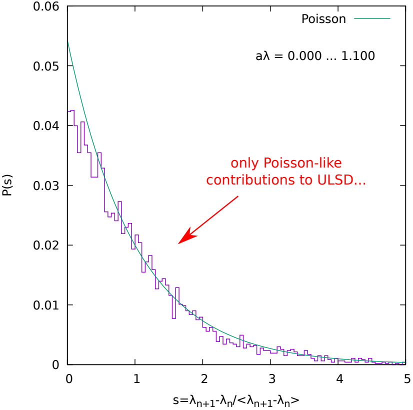
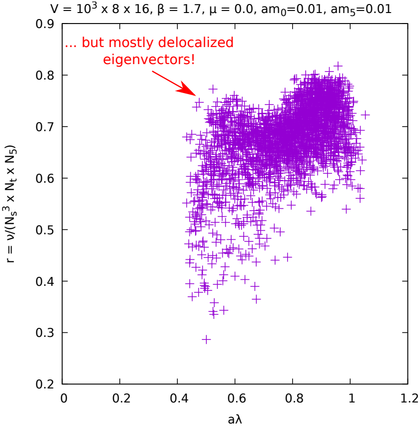


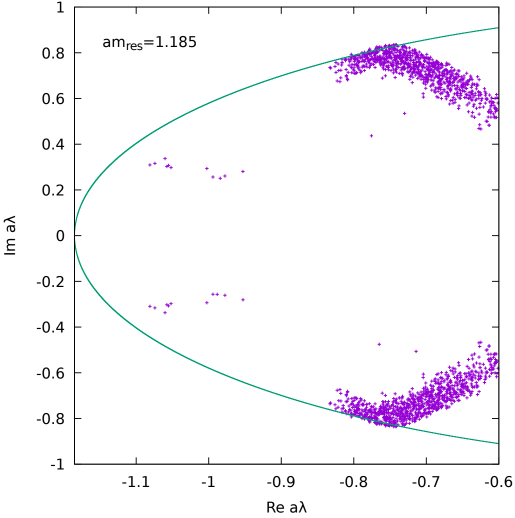


finite size effect?

$V = 16^3 \times 4 \times 8$, $\beta = 2.6$, $am_q = 0.01$, $am_5 = 0.01$

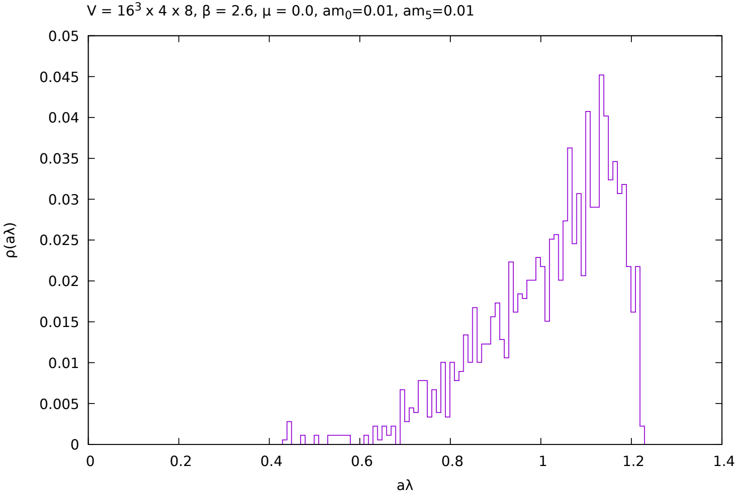
$N_f = 2$ dyn. DW-Quarks, Symanzik imp.

$$\langle P \rangle = -0.5519(1)$$
$$\langle z_2 \rangle = 0.0280(3)$$
$$\langle l \rangle = 0.5598(41)$$
$$\langle \overline{\psi}\psi \rangle = -0.1268(2)$$



$V = 16^3 \times 4 \times 8,\ \beta = 2.6,\ am_q = 0.01,\ am_5 = 0.01$

$N_f = 2$ dyn. DW-Quarks, Symanzik imp.



$V = 16^3 \times 4 \times 8,\ \beta = 2.6,\ am_q = 0.01,\ am_5 = 0.01$

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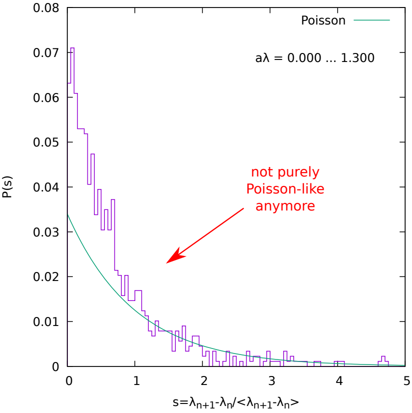
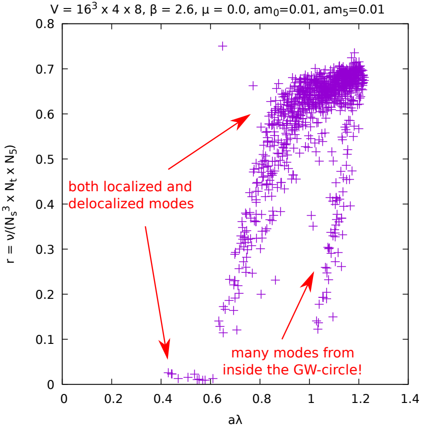


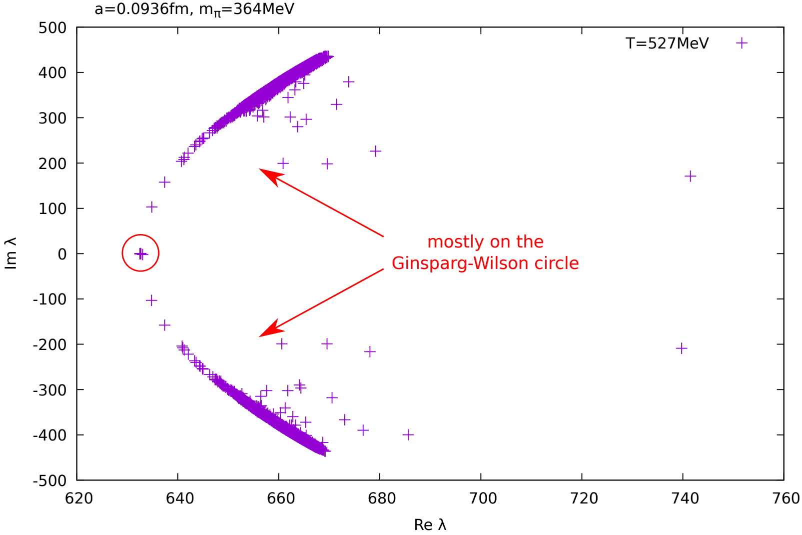


Outline

The Anderson transition in QCD: Preliminary results

- $N_f = 2 + 1 + 1$ Twisted mass sea quarks
- Overlap valence quarks
- $V = 24^3 \times 4$
- $a = 0.0936$ fm
- $T = 527$ MeV
- $m_\pi = 364$ MeV

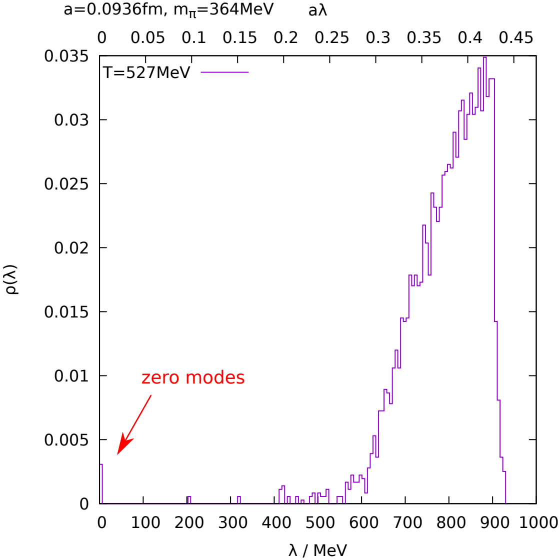
$N_f = 2 + 1 + 1$ Twisted mass sea quarks, Overlap valence quarks



$N_f = 2 + 1 + 1$ Twisted mass sea quarks, Overlap valence quarks

projection to imaginary axis:

$$\lambda_j^{\text{proj}} = \frac{\text{Im}\lambda_j}{1 - \frac{\text{Re}\lambda_j}{2m_q}}$$



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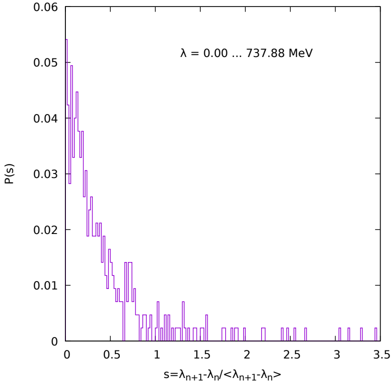
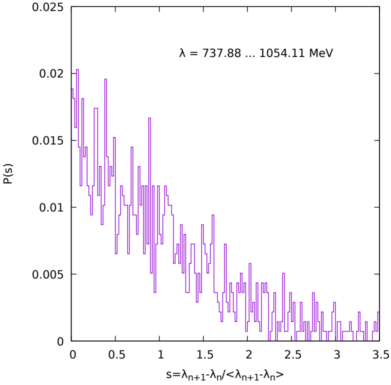
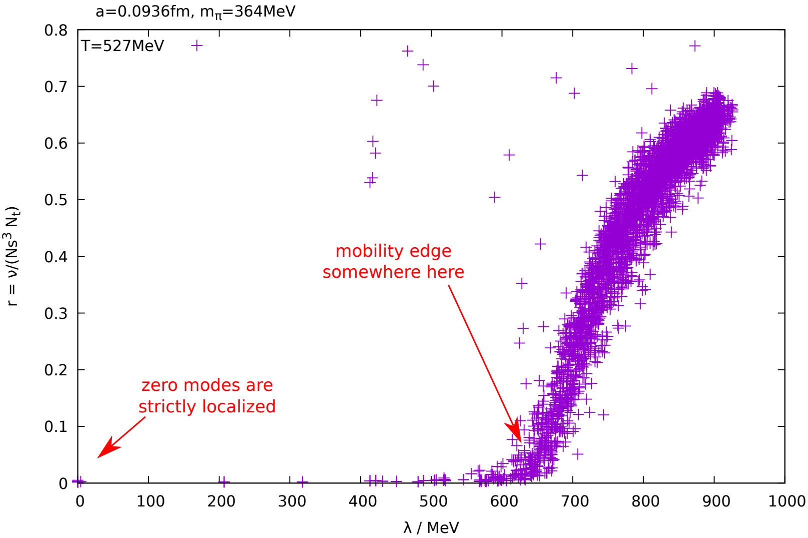




ULS: $s_j = \dfrac{\lambda_{j+1} - \lambda_j}{\langle \lambda_{j+1} - \lambda_j \rangle}$

$P_{\text{Poisson}}(s) \to P_{\text{chGUE}}(s)$?

$N_f = 2 + 1 + 1$ Twisted mass sea quarks, Overlap valence quarks



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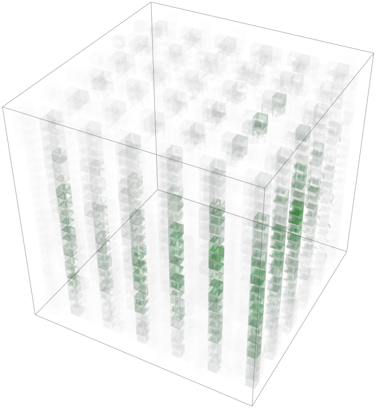
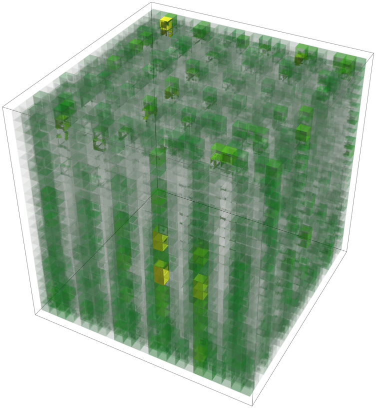


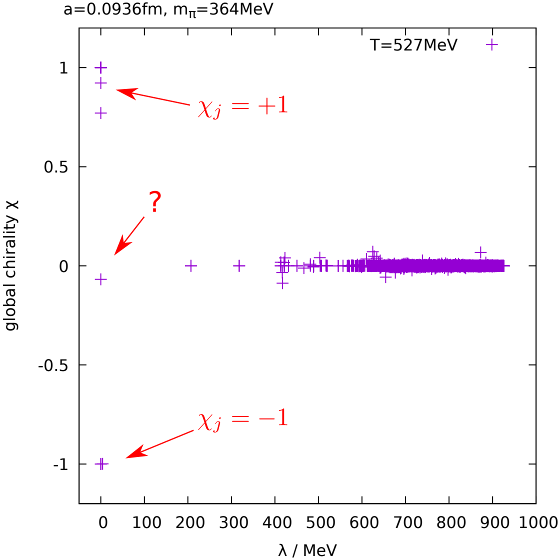
$\lambda_j = 632.75$ MeV



$\lambda_j = 694.93$ MeV

scalar density: $p_j(x) = \langle \psi_j(x) | \psi_j(x) \rangle$

$N_f = 2+1+1$ Twisted mass sea quarks, Overlap valence quarks



pseudoscalar density:

$$p_j^5(x) = \langle \psi_j(x) | \gamma_5 | \psi_j(x) \rangle$$

global chirality:

$$\chi_j = \sum_x p_j^5(x)$$

Outline

Conclusion and outlook

- Many applications for studying quark mode spectra:
  - study mechanism of low eigenmode localization and chiral transition
  - map temperature dependence mobility edge
- Spectral correlations in QCD-like theories:
  - study chiral symmetry at broken γ_5-hermiticity
  - universal behaviour in spectral correlations
- Overlap valence quarks on Twisted Mass sea quarks (tmfT):
  - not restricted to one topological sector
  - yet chirally symmetric fermion operator